

CalcChat[®] and CalcView[®]

PRECALC with LIMITS

5e

Ron Larson

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Ron Larson

The Pennsylvania State University
The Behrend College



Australia • Brazil • Canada • Mexico • Singapore • United Kingdom • United States

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**Precalc with Limits
with CalcChat® and CalcView®
Fifth Edition
Ron Larson**

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*Available at the text companion website *LarsonPrecalculus.com*

Preface

Welcome to *Precalc with Limits with CalcChat® & CalcView®*, Fifth Edition. I am excited to offer you a new edition with more resources than ever that will help you understand and master precalculus with limits. This text includes features and resources that continue to make *Precalc with Limits* a valuable learning tool for students and a trustworthy teaching tool for instructors.


Precalc with Limits provides the clear instruction, precise mathematics, and thorough coverage that you expect for your course. Additionally, this new edition provides you with **free** access to a variety of digital resources:

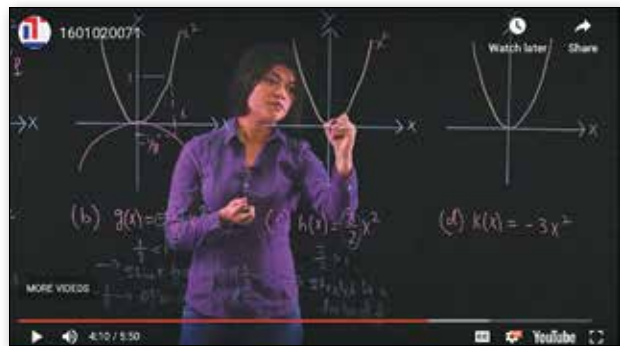
- **GO DIGITAL**—direct access to digital content on your mobile device or computer
- **CalcView.com**—video solutions to selected exercises
- **CalcChat.com**—worked-out solutions to odd-numbered exercises and access to online tutors
- **LarsonPrecalculus.com**—companion website with resources to supplement your learning

These digital resources will help enhance and reinforce your understanding of the material presented in this text and prepare you for future mathematics courses. CalcView® and CalcChat® are also available as free mobile apps.

Features

NEW GO DIGITAL

Scan the QR codes  on the pages of this text to *GO DIGITAL* on your mobile device. This will give you easy access from anywhere to instructional videos, solutions to exercises and Checkpoint problems, Skills Refresher videos, Interactive Activities, and many other resources.



UPDATED CalcView®

The website *CalcView.com* provides video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. The CalcView mobile app is available for free at the Apple® App Store® or Google Play™ store. You can access the video solutions by scanning the QR Code® at the beginning of the Section exercises, or visiting the *CalcView.com* website.

UPDATED CalcChat®


Solutions to all odd-numbered exercises and tests are provided for free at *CalcChat.com*. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. For many years, millions of students have visited my site for help. The CalcChat mobile app is also available as a free download at the Apple® App Store® or Google Play™ store.

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REVISED LarsonPrecalculus.com


All companion website features have been updated based on this revision, including two new features: Skills Refresher and Review & Refresh. Access to these features is free. You can view and listen to worked-out solutions of Checkpoint problems in English or Spanish, explore examples, download data sets, watch lesson videos, and much more.

**NEW Skills Refresher**

This feature directs you to an instructional video where you can review algebra skills needed to master the current topic. Scan the on-page code  or go to *LarsonPrecalculus.com* to access the video.

SKILLS REFRESHER

For a refresher on finding the sum, difference, product, or quotient of two polynomials, watch the video at *LarsonPrecalculus.com*.

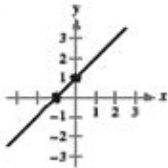
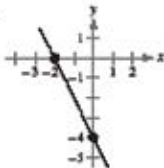
Review & Refresh  Video solutions at LarsonPrecalculus.com

Evaluating an Expression In Exercises 89–92, evaluate the expression. (If not possible, state the reason.)

89. $\frac{5-7}{12-18}$ 90. $\frac{16-6}{6-11}$

91. $\frac{3-3}{4-0}$ 92. $\frac{1-(-1)}{9-9}$

Identifying x - and y -Intercepts In Exercises 93 and 94, identify x - and y -intercepts of the graph.


93.  94. 

Sketching the Graph of an Equation In Exercises 95–98, test for symmetry and graph the equation. Then identify any intercepts.

95. $2x + y = 1$ 96. $3x - y = 7$

97. $y = x^2 + 2$ 98. $y = 2 - x^2$

NEW Review and Refresh

These exercises will help you to reinforce previously learned skills and concepts and to prepare for the next section. View and listen to worked-out solutions of the Review & Refresh exercises in English or Spanish by scanning the code  on the first page of the section exercises or go to *LarsonPrecalculus.com*.

NEW Vocabulary and Concept Check

The Vocabulary and Concept Check appears at the beginning of the exercise set for each section. It includes fill-in-the-blank, matching, or non-computational questions designed to help you learn mathematical terminology and to test basic understanding of the concepts of the section.

NEW Summary and Study Strategies

The “What Did You Learn?” feature is a section-by-section overview that ties the learning objectives from the chapter to the Review Exercises for extra practice. The Study Strategies give concrete ways that you can use to help yourself with your study of mathematics.

REVISED Algebra Help

These notes reinforce or expand upon concepts, help you learn how to study mathematics, address special cases, or show alternative or additional steps to a solution of an example.

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant, and include topics our users have suggested. The exercises have been reorganized and titled so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations. Two new sets of exercises, Vocabulary and Concept Check and Review & Refresh, have been added to help you develop and maintain your skills.

Section Objectives

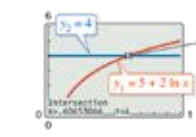
A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

EXAMPLE 7 Solving a Logarithmic Equation

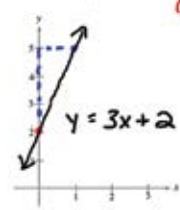
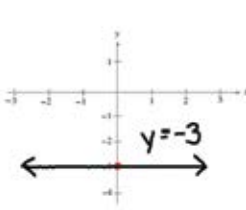
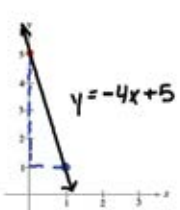
Solve $5 + 2 \ln x = 4$ and approximate the result to three decimal places.

<p>Algebraic Solution</p> $5 + 2 \ln x = 4$ $2 \ln x = -1$ $\ln x = -\frac{1}{2}$ $e^{2 \ln x} = e^{-1/2}$ $x = e^{-1/2}$ $x \approx 0.607$	<p>Graphical Solution</p>  <p>The intersection point is about (0.607, 4).</p> <p>So, the solution is $x \approx 0.607$.</p>
--	--

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Solve $7 + 3 \ln x = 5$ and approximate the result to three decimal places.

Sketch the graph of each linear equation.

<p>a. $y = 3x + 2$</p> <p style="text-align: center;">$y = mx + b$</p> <p style="text-align: center;">↑ slope ↑ y-intercept (0, b)</p> 	<p>b. $y = -3$</p> <p style="text-align: center;">$y = 0x - 3$</p> <p style="text-align: center;">↓ horizontal</p> 	<p>c. $4x + y = 5$</p> <p style="text-align: center;">$-4x \quad -4x$</p> <p style="text-align: center;">$y = -4x + 5$</p> 
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Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at LarsonPrecalculus.com. Scan the on-page code to access the solutions.

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing utilities, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Summarize (Section 2.2)

1. Explain how to use transformations to sketch graphs of polynomial functions (page 124). For an example of sketching transformations of monomial functions, see Example 1.
2. Explain how to apply the Leading Coefficient Test (page 125). For an example of applying the Leading Coefficient Test, see Example 2.
3. Explain how to find real zeros of polynomial functions and use them as sketching aids (page 127). For examples involving finding real zeros of polynomial functions, see Examples 3–5.
4. Explain how to use the Intermediate Value Theorem to help locate real zeros of polynomial functions (page 130). For an example of using the Intermediate Value Theorem, see Example 6.

Summarize

The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool. Use this feature to prepare for a homework assignment, to help you study for an exam, or as a review for previously covered sections.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol \int .

Error Analysis

This exercise presents a sample solution that contains a common error which you are asked to identify.

How Do You See It?

The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at *LarsonPrecalculus.com*.

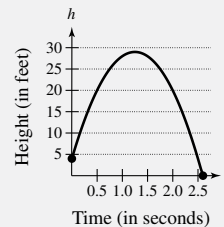
Collaborative Project

You can find these extended group projects at *LarsonPrecalculus.com*. Check your understanding of the chapter concepts by solving in-depth, real-life problems. These collaborative projects provide an interesting and engaging way for you and other students to work together and investigate ideas.



86.

HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



- (a) Explain why h is a function of t .
- (b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.
- (c) Approximate the domain of h .
- (d) Is t a function of h ? Explain.



Built by educators, WebAssign from Cengage is a fully customizable online solution for STEM disciplines. WebAssign includes the flexibility, tools, and content you need to create engaging learning experiences for your students. The patented grading engine provides unparalleled answer evaluation, giving students instant feedback, and insightful analytics highlight exactly where students are struggling. For more information, visit cengage.com/webassign.

Complete Solutions Manual

This manual contains solutions to all exercises from the text, including Chapter Review Exercises and Chapter Tests, and Practice Tests with solutions. The Complete Solutions Manual is available on the Instructor Companion Site.

Cengage Testing Powered by Cognero®

Cengage Testing, Powered by Cognero®, is a flexible online system that allows you to author, edit, and manage test bank content online. You can create multiple versions of your test in an instant and deliver tests from your LMS or exportable PDF or Word docs you print for in-class assessment. Cengage Testing is available online via cengage.com.


Instructor Companion Site

Everything you need for your course in one place! Access and download PowerPoint® presentations, test banks, the solutions manual, and more. This collection of book-specific lecture and class tools is available online via cengage.com.


Test Bank

The test bank contains text-specific multiple-choice and free response test forms and is available online at the Instructor Companion Site.

LarsonPrecalculus.com

In addition to its student resources, LarsonPrecalculus.com also has resources to help instructors. If you wish to challenge your students with multi-step and group projects, you can assign the Section Projects and Collaborative Projects. You can assess the knowledge of your students before and after each chapter using the pre- and post-tests. You can also give your students experience using an online graphing calculator with the Interactive Activities. You can access these features by going to LarsonPrecalculus.com or by scanning the on-page code .

MathGraphs.com

For exercises that ask students to draw on the graph, I have provided **free**, printable graphs at MathGraphs.com. You can access these features by going to MathGraphs.com or by scanning the on-page code  at the beginning of the section exercises, review exercises, or tests.



Prepare for class with confidence using WebAssign from Cengage. This online learning platform, which includes an interactive eBook, fuels practice, so that you truly absorb what you learn and prepare better for tests. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign! Ask your instructor today how you can get access to WebAssign, or learn about self-study options at cengage.com/webassign.


Student Study Guide and Solutions Manual

This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter Tests, and Cumulative Tests. It also contains Practice Tests. For more information on how to access this digital resource, go to cengage.com


Note-Taking Guide

This is an innovative study aid, in the form of a notebook organizer, that helps students develop a section-by-section summary of key concepts. For more information on how to access this digital resource, go to cengage.com


LarsonPrecalculus.com

Of the many features at this website, students have told me that the videos are the most helpful. You can watch lesson videos by Dana Mosely as he explains various mathematical concepts. Other helpful features are the data downloads (editable spreadsheets so you do not have to enter the data), video solutions of the Checkpoint problems in English or Spanish, and the Student Success Organizer. The Student Success Organizer will help you organize the important concepts of each section using chapter outlines. You can access these features by going to LarsonPrecalculus.com or by scanning the on-page code .


CalcChat.com

This website provides free step-by-step solutions to all odd-numbered exercises and tests. Additionally, you can chat with a tutor, at no charge, during the hours posted at the site. You can access the solutions by going to CalcChat.com or by scanning the on-page code  on the first page of the section exercises, review exercises, or tests.

CalcView.com

This website has video solutions of selected exercises. Watch instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. You can access the videos by going to CalcView.com or by scanning the on-page code  on the first page of the section exercises, review exercises, or tests.

MathGraphs.com

For exercises that ask you to draw on the graph, I have provided **free**, printable graphs at MathGraphs.com. You can access the printable graphs by going to MathGraphs.com or by scanning the on-page code  on the first page of the section exercises, review exercises, or tests.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades, I have received many useful comments from both instructors and students, and I value these comments very highly.

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1

Functions and Their Graphs



- 1.1 Rectangular Coordinates
- 1.2 Graphs of Equations
- 1.3 Linear Equations in Two Variables
- 1.4 Functions
- 1.5 Analyzing Graphs of Functions
- 1.6 A Library of Parent Functions
- 1.7 Transformations of Functions
- 1.8 Combinations of Functions: Composite Functions
- 1.9 Inverse Functions
- 1.10 Mathematical Modeling and Variation



Chapter 1 Section 5 Exercise 27 GO

(a)

Zeros: $x = 0, 6$

(b) $f(x) = x^2 - 6x$
 $x^2 - 6x = 0$
 $x(x - 6) = 0$
 $x = 0 \Rightarrow x = 0$
 $x - 6 = 0 \Rightarrow x = 6$

PREV. 17 19 21 23 25 27 29 31 33 35 37 NEXT



1.10 Ocean Temperatures (Exercise 65, p. 102)



1.4 Force of Water (Exercise 64, p. 46)

1.1 Rectangular Coordinates



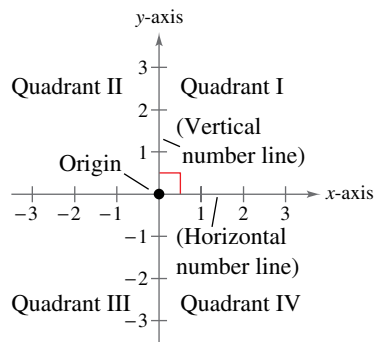
The Cartesian plane can help you visualize relationships between two variables. For example, in Exercise 44 on page 9, given how far north and west one city is from another, plotting points to represent the cities can help you visualize these distances and determine the flying distance between the cities.

- Plot points in the Cartesian plane.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Translate points in the plane.

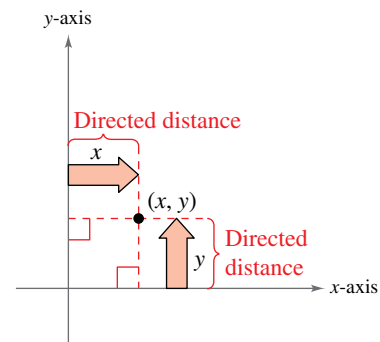
The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system**, or the **Cartesian plane**, named after the French mathematician René Descartes (1596–1650).

Two real number lines intersecting at right angles form the Cartesian plane, as shown in Figure 1.1. The horizontal real number line is usually called the **x-axis**, and the vertical real number line is usually called the **y-axis**. The point of intersection of these two axes is the **origin**, and the two axes divide the plane into four **quadrants**.

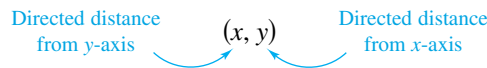


The Cartesian plane
Figure 1.1



Ordered pair (x, y)
Figure 1.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y , called **coordinates** of the point. The **x-coordinate** represents the directed distance from the y-axis to the point, and the **y-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.



The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. Plot the other four points in a similar way, as shown in Figure 1.3.

✓ **Checkpoint** [Audio-video solution in English & Spanish at LarsonPrecalculus.com](#)

Plot the points $(-3, 2)$, $(4, -2)$, $(3, 1)$, $(0, -2)$, and $(-1, -2)$.

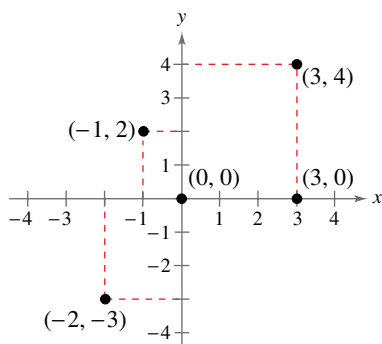


Figure 1.3

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

In the next example, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot**.

EXAMPLE 2 Sketching a Scatter Plot

The table shows the numbers N (in millions) of AT&T wireless subscribers from 2013 through 2018, where t represents the year. Sketch a scatter plot of the data. (Source: AT&T Inc.)

Year, t	2013	2014	2015	2016	2017	2018
Subscribers, N	110	121	129	134	141	153

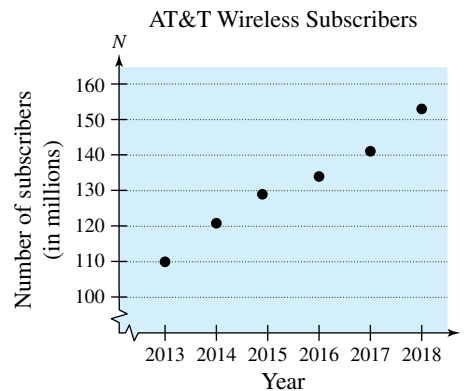
Spreadsheet at LarsonPrecalculus.com

Solution

Before sketching the scatter plot, represent each pair of values in the table by an ordered pair (t, N) , as shown below.

$(2013, 110), (2014, 121), (2015, 129), (2016, 134), (2017, 141), (2018, 153)$

To sketch the scatter plot, first draw a vertical axis to represent the number of subscribers (in millions) and a horizontal axis to represent the year. Then plot a point for each ordered pair, as shown in the figure below. In the scatter plot, the break in the t -axis indicates omission of the numbers less than 2013, and the break in the N -axis indicates omission of the numbers less than 100 million. Also, the scatter plot shows that the number of subscribers has increased each year since 2013.



✓ **Checkpoint** ▶ [Audio-video solution in English & Spanish at LarsonPrecalculus.com](#)

The table shows the numbers N of Costco stores from 2014 through 2019, where t represents the year. Sketch a scatter plot of the data. (Source: Costco Wholesale Corp.)

Year, t	2014	2015	2016	2017	2018	2019
Stores, N	663	686	715	741	762	782

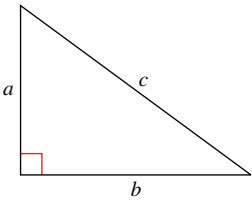
Spreadsheet at LarsonPrecalculus.com

Another way to make the scatter plot in Example 2 is to let $t = 1$ represent the year 2013. In this scatter plot, the horizontal axis does not have a break, and the labels for the tick marks are 1 through 6 (instead of 2013 through 2018).



TECHNOLOGY

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph or a line graph. Use a graphing utility to represent the data given in Example 2 graphically.



The Pythagorean Theorem:
 $a^2 + b^2 = c^2$

Figure 1.4

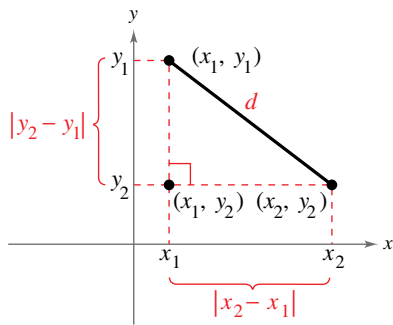


Figure 1.5

The Distance Formula

Before developing the Distance Formula, recall from the **Pythagorean Theorem** that, for a right triangle with hypotenuse of length c and sides of lengths a and b , you have

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$

as shown in Figure 1.4. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Consider two points (x_1, y_1) and (x_2, y_2) that do not lie on the same horizontal or vertical line in the plane. With these two points, you can form a right triangle (see Figure 1.5). To determine the distance d between these two points, note that the length of the vertical side of the triangle is $|y_2 - y_1|$ and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 \quad \text{Pythagorean Theorem}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} \quad \text{Distance } d \text{ must be positive.}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Property of exponents}$$

This result is the **Distance Formula**. Note that for the special case in which the two points lie on the same horizontal or vertical line, the Distance Formula still works. (See Exercise 62.)

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

EXAMPLE 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

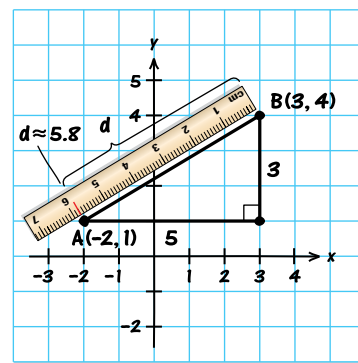
So, the distance between the points is about 5.83 units.

Check

$$\begin{aligned} d^2 &\stackrel{?}{=} 5^2 + 3^2 && \text{Pythagorean Theorem} \\ (\sqrt{34})^2 &\stackrel{?}{=} 5^2 + 3^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B . Then use a centimeter ruler to measure the length of the segment.



The line segment measures about 5.8 centimeters. So, the distance between the points is about 5.8 units.

Checkpoint Audio-video solution in English & Spanish at LarsonPrecalculus.com

Find the distance between the points $(3, 1)$ and $(-3, 0)$.

When the Distance Formula is used, it does not matter which point is (x_1, y_1) and which is (x_2, y_2) , because the result will be the same. For instance, in Example 3, let $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 1)$. Then

$$d = \sqrt{(-2 - 3)^2 + (1 - 4)^2} = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34} \approx 5.83.$$

EXAMPLE 4 Verifying a Right Triangle

Show that the points $(2, 1)$, $(4, 0)$, and $(5, 7)$ are vertices of a right triangle.

Solution

The three points are plotted in Figure 1.6. Use the Distance Formula to find the lengths of the three sides.

$$d_1 = \sqrt{(5 - 2)^2 + (7 - 1)^2} = \sqrt{9 + 36} = \sqrt{45}$$

$$d_2 = \sqrt{(4 - 2)^2 + (0 - 1)^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$d_3 = \sqrt{(5 - 4)^2 + (7 - 0)^2} = \sqrt{1 + 49} = \sqrt{50}$$

Using the converse of the Pythagorean Theorem and the fact that

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude that the triangle is a right triangle.

✓ Checkpoint [▶ Audio-video solution in English & Spanish at LarsonPrecalculus.com](#)

Show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle.

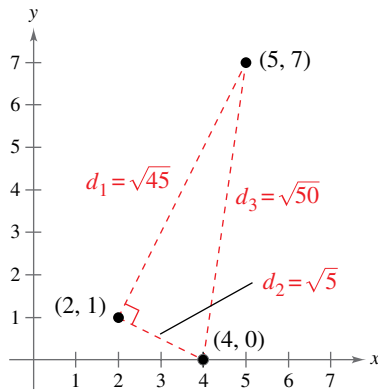


Figure 1.6

EXAMPLE 5 Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. A wide receiver catches the pass on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.7. How long is the pass?

Solution

The length of the pass is the distance between the points $(40, 28)$ and $(20, 5)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{(40 - 20)^2 + (28 - 5)^2} \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{20^2 + 23^2} \quad \text{Simplify.}$$

$$= \sqrt{400 + 529} \quad \text{Simplify.}$$

$$= \sqrt{929} \quad \text{Simplify.}$$

$$\approx 30 \quad \text{Use a calculator.}$$

So, the pass is about 30 yards long.

✓ Checkpoint [▶ Audio-video solution in English & Spanish at LarsonPrecalculus.com](#)

A football quarterback throws a pass from the 10-yard line, 10 yards from the sideline. A wide receiver catches the pass on the 32-yard line, 25 yards from the same sideline. How long is the pass?

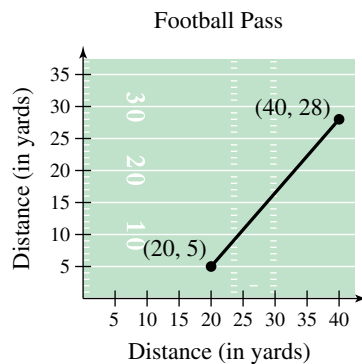


Figure 1.7

In Example 5, the horizontal and vertical scales do not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you may place the coordinate system in any way that helps you solve the problem.



Scan the QR code to access digital content available for this page.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula**.

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 110.

EXAMPLE 6 Finding the Midpoint of a Line Segment

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

Solution Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2} \right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= (2, 0) && \text{Simplify.} \end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure 1.8.

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Find the midpoint of the line segment joining the points $(-2, 8)$ and $(4, -10)$. ■

EXAMPLE 7 Estimating Annual Revenues

Microsoft Corp. had annual revenues of about \$96.7 billion in 2017 and about \$125.8 billion in 2019. Estimate the revenues in 2018. (Source: Microsoft Corp.)

Solution One way to solve this problem is to assume that the revenues followed a *linear* pattern. Then, to estimate the 2018 revenues, find the midpoint of the line segment connecting the points $(2017, 96.7)$ and $(2019, 125.8)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) && \text{Midpoint Formula} \\ &= \left(\frac{2017 + 2019}{2}, \frac{96.7 + 125.8}{2} \right) && \text{Substitute for } x_1, x_2, y_1, \text{ and } y_2. \\ &= (2018, 111.25) && \text{Simplify.} \end{aligned}$$

So, you would estimate the 2018 revenues to have been about \$111.25 billion, as shown in Figure 1.9. (The actual 2018 revenues were about \$110.36 billion.)

✓ Checkpoint  [Audio-video solution in English & Spanish at LarsonPrecalculus.com](#)

The Proctor & Gamble Co. had annual sales of about \$65.1 billion in 2017 and about \$67.7 billion in 2019. Estimate the sales in 2018. (Source: Proctor & Gamble Co.) ■

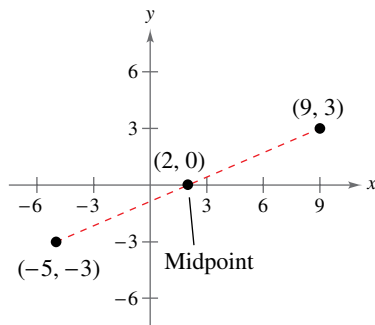


Figure 1.8

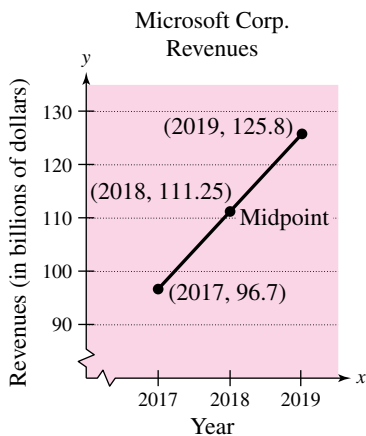
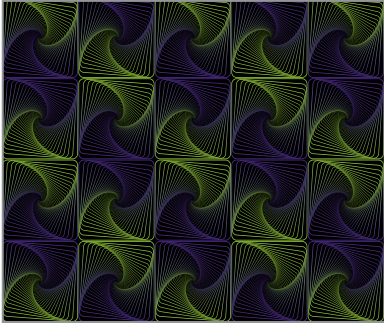


Figure 1.9





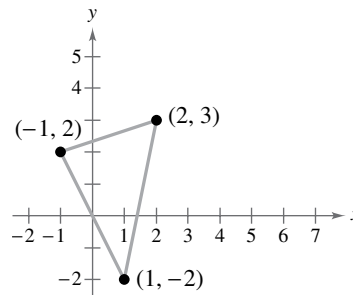
Application

Much of computer graphics, including the computer-generated tessellation shown at the left, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

EXAMPLE 8 Translating Points in the Plane

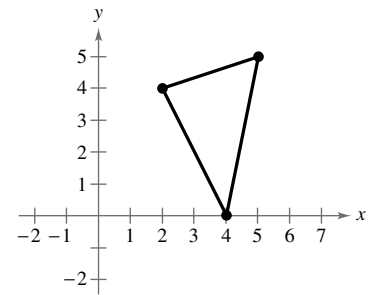
▶▶▶ See LarsonPrecalculus.com for an interactive version of this type of example.

The triangle in Figure 1.10 has vertices at $(-1, 2)$, $(1, -2)$, and $(2, 3)$. Shift the triangle three units to the right and two units up. What are the coordinates of the vertices of the shifted triangle (see Figure 1.11)?



Original triangle

Figure 1.10



Shifted triangle

Figure 1.11

Solution To shift the vertices three units to the right, add 3 to each of the x -coordinates. To shift the vertices two units up, add 2 to each of the y -coordinates.

Original Point	Translated Point
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -2)$	$(1 + 3, -2 + 2) = (4, 0)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

✓ **Checkpoint** ▶ [Audio-video solution in English & Spanish at LarsonPrecalculus.com](http://LarsonPrecalculus.com)

The parallelogram in Figure 1.12 has vertices at $(1, 4)$, $(1, 0)$, $(3, 2)$, and $(3, 6)$. Shift the parallelogram two units to the left and four units down. What are the coordinates of the vertices of the shifted parallelogram?

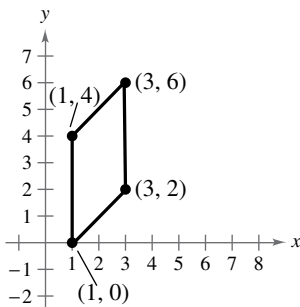


Figure 1.12

The figures in Example 8 were not really essential to the solution. Nevertheless, you should develop the habit of including sketches with your solutions because they serve as useful problem-solving tools.

Summarize (Section 1.1)

1. Describe the Cartesian plane (*page 2*). For examples of plotting points in the Cartesian plane, see Examples 1 and 2.
2. State the Distance Formula (*page 4*). For examples of using the Distance Formula to find the distance between two points, see Examples 3–5.
3. State the Midpoint Formula (*page 6*). For examples of using the Midpoint Formula to find the midpoint of a line segment, see Examples 6 and 7.
4. Describe how to translate points in the plane (*page 7*). For an example of translating points in the plane, see Example 8.



1.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.



Vocabulary and Concept Check

In Exercises 1 and 2, fill in the blanks.

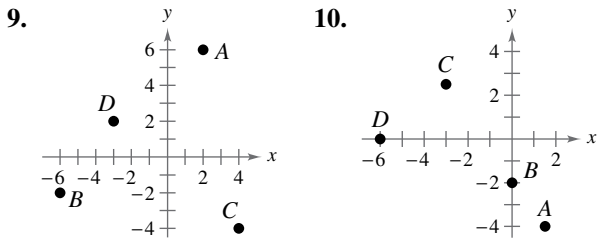
1. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the _____ plane.
2. Finding the average values of the respective coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the _____.

In Exercises 3–8, match each term with its definition.

- | | |
|--------------------|--|
| 3. x -axis | (a) point of intersection of vertical axis and horizontal axis |
| 4. y -axis | (b) directed distance from the x -axis |
| 5. origin | (c) horizontal real number line |
| 6. quadrants | (d) four regions of the coordinate plane |
| 7. x -coordinate | (e) directed distance from the y -axis |
| 8. y -coordinate | (f) vertical real number line |

Skills and Applications

Approximating Coordinates of Points In Exercises 9 and 10, approximate the coordinates of the points.



Plotting Points in the Cartesian Plane In Exercises 11 and 12, plot the points.

11. $(2, 4)$, $(3, -1)$, $(-6, 2)$, $(-4, 0)$, $(-1, -8)$, $(1.5, -3.5)$
12. $(1, -5)$, $(-2, -7)$, $(3, 3)$, $(-2, 4)$, $(0, 5)$, $(\frac{2}{3}, \frac{5}{2})$

Finding the Coordinates of a Point In Exercises 13 and 14, find the coordinates of the point.

13. The point is three units to the left of the y -axis and four units above the x -axis.
14. The point is on the x -axis and 12 units to the left of the y -axis.

Determining Quadrant(s) for a Point In Exercises 15–20, determine the quadrant(s) in which (x, y) could be located.

- | | |
|---|-------------------------|
| 15. $x > 0$ and $y < 0$ | 16. $x < 0$ and $y < 0$ |
| 17. $x = -4$ and $y > 0$ | 18. $x < 0$ and $y = 7$ |
| 19. $x + y = 0$, $x \neq 0$, $y \neq 0$ | 20. $xy > 0$ |

Sketching a Scatter Plot In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.

21. The table shows the number y of Dollar General stores for each year x from 2012 through 2018. (Source: Dollar General Corporation)

Year, x	Number of Stores, y
2012	10,506
2013	11,132
2014	11,789
2015	12,483
2016	13,320
2017	14,609
2018	15,472

22. The table shows the annual revenues y (in billions of dollars) for Amazon.com for each year x from 2011 through 2018. (Source: Amazon.com)

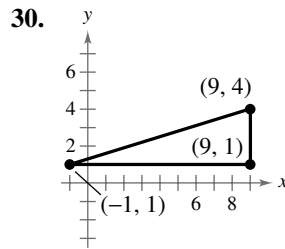
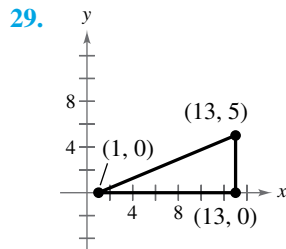
Month, x	Annual Revenue, y
2011	48.1
2012	61.1
2013	74.5
2014	89.0
2015	107.0
2016	136.0
2017	177.9
2018	232.9

A blue exercise number indicates that a video solution can be seen at CalcView.com.

Finding a Distance In Exercises 23–28, find the distance between the points.

23. $(-2, 6)$, $(3, -6)$ 24. $(8, 5)$, $(0, 20)$
 25. $(1, 4)$, $(-5, -1)$ 26. $(1, 3)$, $(3, -2)$
 27. $(\frac{1}{2}, \frac{4}{3})$, $(2, -1)$ 28. $(9.5, -2.6)$, $(-3.9, 8.2)$

Verifying a Right Triangle In Exercises 29 and 30, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.



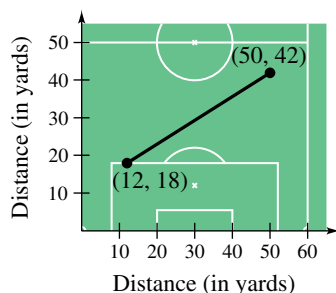
Verifying a Polygon In Exercises 31–34, show that the points form the vertices of the polygon.

31. Right triangle: $(4, 0)$, $(2, 1)$, $(-1, -5)$
 32. Right triangle: $(-1, 3)$, $(3, 5)$, $(5, 1)$
 33. Isosceles triangle: $(1, -3)$, $(3, 2)$, $(-2, 4)$
 34. Isosceles triangle: $(2, 3)$, $(4, 9)$, $(-2, 7)$

Plotting, Distance, and Midpoint In Exercises 35–42, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

35. $(6, -3)$, $(6, 5)$ 36. $(1, 4)$, $(8, 4)$
 37. $(1, 1)$, $(9, 7)$ 38. $(1, 12)$, $(6, 0)$
 39. $(-1, 2)$, $(5, 4)$
 40. $(2, 10)$, $(10, 2)$
 41. $(-16.8, 12.3)$, $(5.6, 4.9)$
 42. $(\frac{1}{2}, 1)$, $(-\frac{5}{2}, \frac{4}{3})$

43. **Sports** A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. A teammate who is 42 yards from the same endline and 50 yards from the same sideline receives the pass. (See figure.) How long is the pass?



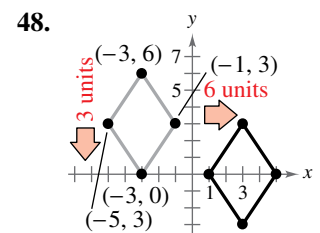
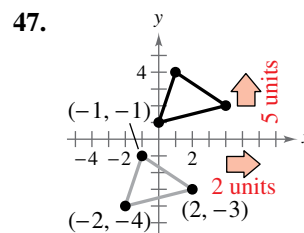
44. Flying Distance

An airplane flies from Naples, Italy, in a straight line to Rome, Italy, which is about 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?



45. **Sales** Walmart had sales of \$485.9 billion in 2016 and \$514.4 billion in 2018. Use the Midpoint Formula to estimate the sales in 2017. Assume that the sales followed a linear pattern. (Source: Walmart, Inc.)
 46. **Earnings per Share** The earnings per share for Facebook, Inc. were \$6.16 in 2017 and \$7.57 in 2018. Use the Midpoint Formula to estimate the earnings per share in 2019. Assume that the earnings per share followed a linear pattern. (Source: Facebook, Inc.)

Translating Points in the Plane In Exercises 47–50, find the coordinates of the vertices of the polygon after the given translation to a new position in the plane.



49. Original coordinates of vertices: $(-7, -2)$, $(-2, 2)$, $(-2, -4)$, $(-7, -4)$
 Shift: eight units up, four units to the right
 50. Original coordinates of vertices: $(5, 8)$, $(3, 6)$, $(7, 6)$
 Shift: 6 units down, 10 units to the left

Exploring the Concepts

True or False? In Exercises 51–54, determine whether the statement is true or false. Justify your answer.

51. If the point (x, y) is in Quadrant II, then the point $(2x, -3y)$ is in Quadrant III.
 52. To divide a line segment into 16 equal parts, you have to use the Midpoint Formula 16 times.
 53. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.
 54. If four points represent the vertices of a polygon, and the four side lengths are equal, then the polygon must be a square.
 55. **Think About It** What is the y -coordinate of any point on the x -axis? What is the x -coordinate of any point on the y -axis?